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14. ABSTRACT

There are many problems of scientific interest that are not tractable on any foreseeable classical computer. Quantum computers have the potential of exploiting non-classical features like quantum entanglement and phase coherence which can exponentially speed-up the computational algorithm. Quantum algorithms have been developed to study the evolution of solitons in the Korteveg-de Vries equation and the Nonlinear Schrodinger equation. These nonlinear equations have known exact analytic solutions to which the quantum algorithm solutions have been compared - with excellent agreement. Vector soliton propagation down birefringment media have also been considered as well as soliton turbulence and the corresponding power spectra. It has been found that two on-site qubits per spatial node is sufficient to model a scalar continuum field. The particular choice and sequence of unitary collision and streaming opereators dictate the final continuum form of the partial differential equation.

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AFOSR FINAL REPORT

"TYPE-II QUANTUM ALGORITHMS for SOLITONS"

February, 2004

George Vahala
William & Mary

During the period of this grant, we have been working on the development of Quantum Algorithms for nonlinear physical systems, in collaboration with Dr. Jeff Yepez (Hanscom Field) and Dr. Linda Vahala (Old Dominion University). In particular, quantum algorithms have been developed for

(a) solitons

"Quantum Lattice Gas Representation of Some Classical Solitons"
G. Vahala, J. Yepez and L. Vahala
Phys. Lett. **A310**, 187-196 (2003)

"Quantum lattice gas representation for vector solitons"
G. Vahala, L. Vahala, and J. Yepez
SPIE Conf. Proc. **5105,** 273 – 281 (2003)

"Inelastic Vector Soliton Collisions: A Quantum Lattice Gas Representation"
G. Vahala, L. Vahala and J. Yepez
Phil. Trans.. Roy Soc. London (to be published)

"Quantum Lattice Representation of Dark Solitons"
G. Vahala, L. Vahala, and J. Yepez
SPIE Conf. Proc, submitted (2004)

(b) 1D MHD-Burgers equation

"Lattice Boltzmann and Quantum Lattice Gas Representations of One-Dimensional Magnetohydrodynamic Turbulence"
L. Vahala, G. Vahala and J. Yepez
Phys. Lett. **A306**, 227-234 (2003)

We have predominantly concentrated on soliton research since exact solutions are known for KdV and both the scalar and vector nonlinear Schrodinger equation (NLS) as these will provide a stringent test on our quantum algorithms. The spatial dimension is discretized into a set of spatial nodes. For modeling either the KdV equation or the scalar and vector NLSeach scalar field component we require 2 qubits at each lattice site. The on-site qubits are entangled by the unitary collision operator and this entanglement is spread throughout the system by unitary streaming. In particular, the KdV equation is modeled by the tensor product of the on-site unitary collision matrix

$$\hat{U}_{KdV} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

with the collide-stream algorithm

$$|\psi(t+\Delta t)\rangle = \hat{S}_1 \hat{C}^+ \hat{S}_2^T C.\hat{S}_1^T \hat{C}^+ \hat{S}_2 \hat{C}, \hat{S}_1^T \hat{C}\ \hat{S}_2 \hat{C}^+ . \hat{S}_1 \hat{C} \hat{S}_2^T C^+ |\psi(t)\rangle$$

where $\hat{S}_1$ is the global streaming operator on qubit-1 to the neighboring lattice site while the transpose streaming operator on qubit-2 is $\hat{S}_2$. $\hat{C}$ is the tensor product of the on-site collision matrix $\hat{U}_{KdV}$ and $\hat{C}^+$ is the adjoint operator. In the continuum limit, the resulting partial differential equation is the KdV equation

$$\frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi}{\partial x} + \frac{1}{2}\frac{\partial^3 \psi}{\partial x^3} = 0$$

after the phase transformation is introduced to yield the nonlinear "potential" term

$$\psi \rightarrow \exp[iV\Delta t]\psi \ , \text{ with } \ V = i\frac{\partial \psi}{\partial x}$$

The specific streaming sequence is required to eliminate the standard diffusive/dispersive $\partial^2/\partial x^2$ term and thus give the required leading order linear term of the KdV-equation $\partial^3/\partial x^3$.

To recover the scalar NLS we now entangle the on-site two qubits by the unitary collision operator

$$\hat{U}_{NLS} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and utilize the global collide-stream algorithm

$$|\psi(t+\Delta t)\rangle = \hat{S}_2^T \hat{C}\ \hat{S}_2 \hat{C}\ .\hat{S}_2^T \hat{C} \hat{S}_2 \hat{C}\ .\ \hat{S}_1^T \hat{C}\ \hat{S}_1 \hat{C}\ .\hat{S}_1^T \hat{C} \hat{S}_1 \hat{C}\ |\psi(t)\rangle$$

The accuracy of the finite quantum difference algorithm becomes second order by symmetrizing the collide-stream algorithm on each on-site qubit. The nonlinear potential term is recovered by the phase transformation

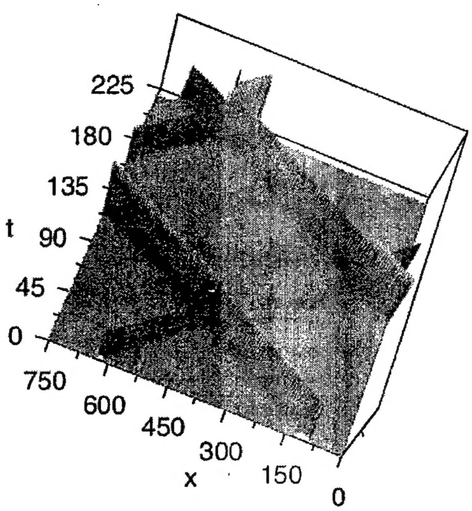
$$\psi \rightarrow \exp[iV\Delta t]\psi \ , \text{ with } \ V = |\psi|^2$$

to yield the cubic NLS in the continuum limit

$$i\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + |\psi|^2 \psi = 0$$

These algorithms are readily extended to consider the coupling of the polarizations due to the birefringent medium. In this case we have two coupled NLS equations, requiring two qubits/node for each polarization. The coupling between the two polarizations is achieved by the appropriate coupling phase transformation that now couples all four on-site qubits. An interesting exactly soluble example is the inelastic collision of vector Manakov solitons in which for one of the polarizations a soliton is destroyed. It reforms following a second collision

ψ_1 - polarization



ψ_2 – polarization